STUDENT'S NAME: ________________________________

TEACHER'S NAME: ________________________________

PRO PATRIA

2019

TRIAL HIGHER SCHOOL CERTIFICATE **EXAMINATION**

Mathematics Extension 1 Assessment Task 4

Examiners

- Mrs Biczo
- Ms Crancher
- Mr Potaczala
- Ms Tarannum

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference sheet is provided for your use
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II –60 marks (pages 5 – 10)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section 1
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section.
Use the multiple choice answer sheet provided for Questions 1 – 10

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1. Which of the following is an expression for $\int 2\sin^2 x\,dx$?

A. $x+\frac{1}{2}\sin 2x+c$

B. $x-\frac{1}{2}\sin 2x+c$

C. $x+\sin 2x+c$

D. $x-\sin 2x+c$

2. The general solution for $\cos\theta=-\frac{1}{2}$ is given by which of the following?

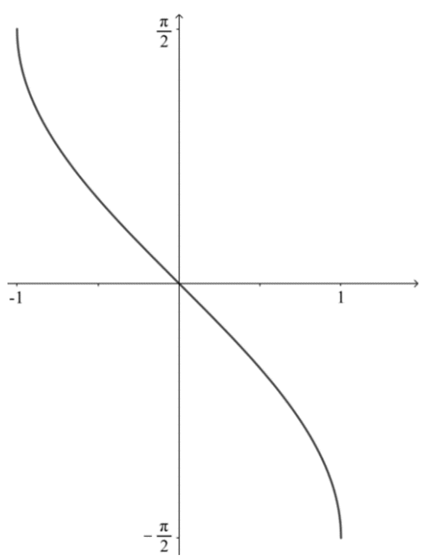
A. $\theta=2n\pi\pm\frac{\pi}{3}$, where n is real

B. $\theta=2n\pi\pm\frac{\pi}{3}$, where n is an integer

C. $\theta=2n\pi\pm\frac{2\pi}{3}$, where n is real

D. $\theta=2n\pi\pm\frac{2\pi}{3}$, where n is an integer

3. Which of the following functions best describes the graph below?



A. $y=\sin^{-1}x$

B. $y=-\sin^{-1}x$

C. $y=\cos^{-1}x$

D. $y=-\cos^{-1}x$

4. What is the inverse of the function $y = x^3 + 2$?

A. $y = x^3 - 2$

B. $y = \pm\sqrt[3]{x-2}$

C. $x = y^3 - 2$

D. $y = \sqrt[3]{x-2}$

5. What is the coefficient of x^3 in the expansion of $(2x-3)^4$?

A. –96 **B.** 96 **C.** –216 **D.** 216

6. Suppose that X is the number of female children born into a family. Given that the distribution of X is binomial, with a probability of success of $0 \cdot 48$, what is the probability that a family with five children will have two female children?

A. $(0 \cdot 48)^2$

B. $(0 \cdot 48)^2 (0.52)^3$

C. ${}^5C_2 (0 \cdot 48)^2 (0.52)^3$

D. ${}^5C_2 (0 \cdot 52)^2 (0 \cdot 48)^3$

7. By considering the binomial expansion:

$$ {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n = (1+x)^n, $$

to show $\sum_{r=0}^{n} {}^nC_r = 2^n$, what must you do?

A. Differentiate both sides of the binomial expansion

B. Substitute $x = 1$ into both sides of the binomial expansion

C. Substitute $x = -1$ into both sides of the binomial expansion

D. Substitute $x = 0$ into both sides of the binomial expansion

8. When the polynomial $P(x) = x^3 + ax^2 + 7$ is divided by $x + 2$, the remainder is 11. What is the value of a?

A. 3 **B.** -3 **C.** -4 **D.** 26

9. What is the solution to the inequality $\frac{x^2 - 4}{2x} < 0$?

A. $x < -2$ and $0 < x < 2$

B. $-4 < x < 0$ and $x > 2$

C. $-2 < x < 0$ and $x > 2$

D. $-2 < x < 0$ and $x > 4$

10. In how many ways can 5 men and 5 women be arranged around a circular table if the women and men are to alternate?

A. 600 **B.** 2 880 **C.** 14 400 **D.** 86 400

End of Section 1

Section 2
60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section
Answer each question in a separate answer booklet.
In questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new answer booklet. **Marks**

(a) Prove that $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\dfrac{\theta}{2}$ **2**

(b) (i) Write $\cos x-\sqrt{3}\sin x$ in the form $r\cos(x+\alpha)$ **2**

(ii) Hence, find the maximum value of $\cos x-\sqrt{3}\sin x$ and the first positive value of x for which this occurs. **2**

(c) From the top of a vertical tower, of height h, the angle of depression to a man M, standing due south of the base O of the tower, is $42°$. From the top of the tower, another man G is observed with angle of depression $32°$. The men are standing 500 metres apart, with G due east of M. Find the height h of the tower, to the nearest metre. **3**

(d) The curves $f(x)=\sin x$ and $g(x)=\cos x$ meet at $x=\dfrac{\pi}{4}$. It is given that $f'\left(\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{2}}$ and $g'\left(\dfrac{\pi}{4}\right)=\dfrac{-1}{\sqrt{2}}$. Calculate the acute angle between the curves at $x=\dfrac{\pi}{4}$. Give your answer in radians to two decimal places. **2**

Question 11 is continued on the next page

Question 11 continued

(e) (i) Show that $\tan(x+h)-\tan x=\dfrac{\sin \mathrm{h}}{\cos(x+h)\cos x}$ **2**

(ii) Hence, using the first principles definition of the derivative, **2**

$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, show that if $f(x)=\tan x$, then $f'(x)=\sec^2 x$.

Question 12 is on the next page

Question 12 (15 marks) Start a new answer booklet. **Marks**

(a) Consider the function $y = \tan^{-1}\frac{1}{x}$, where $x \neq 0$:

(i) Find $\frac{dy}{dx}$ **2**

(ii) Show that $\frac{d}{dx}\left(\tan^{-1}x + \tan^{-1}\frac{1}{x}\right) = 0$ **1**

(iii) Hence, or otherwise, sketch $y = \tan^{-1}x + \tan^{-1}\frac{1}{x}$ for $x > 0$ **1**

(b) Using the substitution $u = \sqrt{x}$, find $\int_1^9 \frac{dx}{x+\sqrt{x}}$. **3**

(c) Show that the polynomial $P(x) = 2x^3 - 5x^2 - 9x + 18$ has a root x such that $1 < x < 2$. **2**

(d) In a town, the annual growth rate of the population N is given by $\frac{dN}{dt} = k(N - 125)$, where k is a constant.

(i) Show that $N = 125 + Ae^{kt}$ is a solution to $\frac{dN}{dt} = k(N - 125)$ **2**

(ii) If the initial population was 25,650, find k given the population was 31,100 after 5 years. Round your answer to three decimal places. **2**

(e) Sketch the graph $y = \frac{x^2}{4 - x^2}$, clearly showing any asymptotes or intercepts with the co-ordinate axes. Your graph should be at least a quarter of a page. **2**

Question 13 (15 marks) Start a new answer booklet. **Marks**

(a) Prove that for all positive integers n, $9^{n+2} - 4^n$ is divisible by 5. **3**

(b) (i) By using the binomial expansion, show that **2**

$$(a+b)^n - (a-b)^n = 2\binom{n}{1}a^{n-1}b + 2\binom{n}{3}a^{n-3}b^3 + 2\binom{n}{5}a^{n-5}b^5 + \ldots$$

(ii) What is the last term in the expansion when n is even? **1**

Give your answer in simplest form.

(c) A fair six-sided die is randomly tossed n times.

(i) Suppose $0 \le r \le n$.

What is the probability that exactly r 'sixes' appear in the uppermost position? **2**

(ii) **By using the result of part (b), or otherwise**, show that the probability that an odd number of 'sixes' appears is $\frac{1}{2}\left\{1-\left(\frac{2}{3}\right)^n\right\}$ **2**

(d) Let A be the point $(-3,7)$ and let B be the point $(1,6)$. Find the coordinates of the point P which divides the interval AB externally in the ratio 1 : 2. **2**

(e) A spherical balloon is being deflated so that the radius decreases at a constant rate of 8 mm/s. Calculate the rate at which the volume is changing at the instant when the radius is 100 mm. **3**

Question 14 (15 marks) Start a new answer booklet. **Marks**

(a) Four women and three men are available for selection in a team. A team of 4 players consists of two men and two women.

(i) How many different teams of four players can be selected? **1**

(ii) Two players are husband and wife and wish to play in the same team. How many different teams can now be selected with the husband and wife on the same team? **1**

(b)

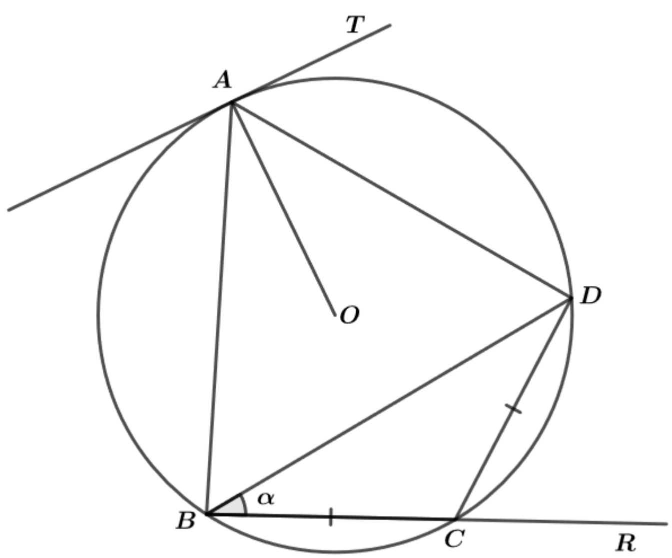


Diagram not to scale

In the diagram above, the points A, B, C and D lie on a circle with centre O. The line TA is a tangent to the circle. The chord BC is produced to R. The interval AO bisects $\angle BAD$ and $BC = CD$. Let $\angle DBC = \alpha$.

(i) Prove that $\angle DCR = 2\alpha$. **1**

(ii) Show that $\angle OAD = \alpha$. **1**

(iii) Prove that $\angle ABC$ is a right angle. **3**

Question 14 is continued on the next page

Question 14 continued

(c) Let α, β and γ be the roots of $3x^3 + 8x^2 - 1 = 0$. **3**

What is the value of $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\gamma}\right)\left(\gamma + \frac{1}{\alpha}\right)$?

(d) $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ are two points on the parabola $x^2 = 4ay$.

(i) Show that the co-ordinates of R, the point of intersection of the normals at P and Q are $\left(-apq\left(p+q\right), a\left(p^2 + pq + q^2 + 2\right)\right)$. **2**

(ii) If $pq = -2$ find the cartesian equation of the locus of R. **3**

End of Examination

Year 12 Trial Higher School Certificate | Extension 1 Mathematics | Examination 2019

Question No. 11 | Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE2 Uses multi-step deductive reasoning in a variety of contexts

PE6 Makes comprehensive use of mathematical language, diagrams & notation for communicating in a wide variety of situations

HE1 Appreciates interrelationships between ideas drawn from different areas of mathematics

HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

Outcome	Solutions	Marking Guidelines
PE2	(a) Let $t=\tan\frac{\theta}{2}$	2 marks : correct solution 1 mark: significant progress toward correct solution

Then LHS $= \dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$

$$= \frac{1+\frac{2t}{1+t^2}-\left(\frac{1-t^2}{1+t^2}\right)}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$

$$= \frac{1+t^2+2t-(1-t^2)}{1+t^2+2t+1-t^2} \quad \text{(multiplying by } t^2\text{)}$$

$$= \frac{2t^2+2t}{2+2t}$$

$$= \frac{2t(t+1)}{2(1+t)}$$

$$= t$$

$$= \tan\frac{\theta}{2}$$

Note: a number of students did not state the expression on the LHS, instead starting from the second line. This can result in loss of marks in the HSC.

Outcome	Solutions	Marking Guidelines
PE2	(b) (i) $\cos x-\sqrt{3}\sin x \equiv r\cos(x+\alpha)$	2 marks : correct solution 1 mark : correct value for r or α

$\equiv r(\cos x\cos\alpha-\sin x\sin\alpha)$ where

$$r=\sqrt{1^2+\left(-\sqrt{3}\right)^2}=2$$

$\cos x-\sqrt{3}\sin x \equiv 2\cos x\cos\alpha-2\sin x\sin\alpha$

Equating like coefficients, $1=2\cos\alpha$, $\sqrt{3}=2\sin\alpha$

$\therefore \sin\alpha=\frac{\sqrt{3}}{2}$, $\cos\alpha=\frac{1}{2}$

As sin positive quadrants 1, 2 & cos positive in quadrants 1, 4 α is in quadrant 1. $\therefore \alpha=\frac{\pi}{3}$

$\therefore \cos x-\sqrt{3}\sin x \equiv 2\cos\left(x+\frac{\pi}{3}\right)$

Outcome	Solutions	Marking Guidelines
PE2, PE6	(ii) Maximum value of $\cos x-\sqrt{3}\sin x$ is when	2 marks : correct answers 1 mark : one correct answer or equivalent

$2\cos\left(x+\frac{\pi}{3}\right)$ is a maximum which occurs when

$\cos\left(x+\frac{\pi}{3}\right)=1$. $\therefore$ maximum value is $2\times1=2$.

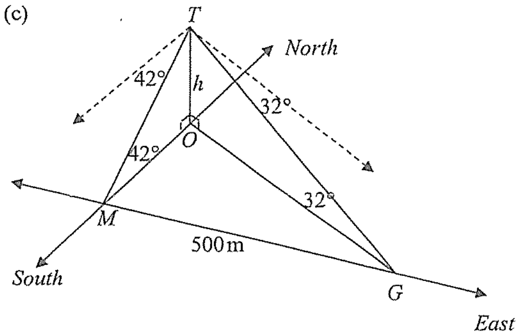
$\cos\left(x+\frac{\pi}{3}\right)=1$ when $x+\frac{\pi}{3}=\ldots, 0, 2\pi, \ldots$

i.e. when $x=\ldots,-\frac{\pi}{3}, 2\pi-\frac{\pi}{3},\ldots$

$\therefore$ maximum value is 2, and the smallest positive value of x for which it occurs is $\frac{5\pi}{3}$.

Note: many students differentiated to find the maximum value. This is not always necessary with trig functions as sin and cos have maximum 1.

Outcome	Solutions	Marking Guidelines
PE2	(c)	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution



Let T be the top of the tower.

In Δ's OTG & OTM, $\cot 32°=\frac{OG}{h}$ & $\cot 42°=\frac{OM}{h}$ respectively

$\therefore OG=h\cot 32°$ & $OM=h\cot 42°$

As G is east of M, $\angle OMG=90°$

$\therefore OM^2+500^2=OG^2$ (Pythagoras)

$h^2\cot^2 42°+500^2=h^2\cot^2 32°$

$500^2=h^2\left(\cot^2 32°-\cot^2 42°\right)$

$\therefore h^2=\dfrac{500^2}{\cot^2 32°-\cot^2 42°}$

$\therefore h=\dfrac{500}{\sqrt{\tan^2 58°-\tan^2 48°}}$ using complementary angle results

$\therefore h=434$ m

Note: Many students did not correctly position the angles of depression, or have $\angle OMG$ as the right angle.

Outcome	Solutions	Marking Guidelines
	(d) Using $\tan\theta=\left\lvert\dfrac{m_1-m_2}{1+m_1m_2}\right\rvert$,	2 marks : correct solution 1 marks : substantial progress towards correct solution

$$\tan\theta=\left|\frac{\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)}{1+\frac{1}{\sqrt{2}}\times\left(-\frac{1}{\sqrt{2}}\right)}\right|$$

PE2

$$\tan\theta = \left|\frac{\sqrt{2}}{\frac{1}{2}}\right|$$

$$\tan\theta = 2\sqrt{2}$$

$$\theta = 1.23^c$$

HE7, HE1

(e) (i)

$$\tan(x+h) - \tan x = \frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}$$

$$= \frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos(x+h)\cos x}$$

$$= \frac{\sin((x+h)-x)}{\cos(x+h)\cos x}$$

$$= \frac{\sin h}{\cos(x+h)\cos x}$$

Marking guidelines: 2 marks : correct solution. 1 marks : substantial progress towards correct solution.

(ii) Given $f(x) = \tan x$ and $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$,

$$f'(x) = \lim_{h\to 0}\frac{\tan(x+h)-\tan(x)}{h},$$

$$= \lim_{h\to 0}\frac{\frac{\sin h}{\cos(x+h)\cos x}}{\cdot h},$$

$$= \lim_{h\to 0}\frac{\sin h}{\cos(x+h)\cos x}\times\frac{1}{h},$$

$$\therefore f(x) = \lim_{h\to 0}\frac{\sin h}{h}\times\frac{1}{\cos(x+h)\cos x},$$

$$= \lim_{h\to 0}\frac{\sin h}{h}\times\lim_{h\to 0}\frac{1}{\cos(x+h)\cos x},$$

$$= 1\times\frac{1}{\cos(x+0)\cos x},$$

$$= \frac{1}{\cos^2 x}$$

$$\therefore f'(x) = \sec^2 x$$

In show that questions, do not leave steps out. To ensure full marks, the substitution should be shown and all steps justified, such as $\lim_{h\to 0}\frac{\sin h}{h} = 1$.

Marking guidelines: 2 marks : correct solution. 1 mark : substantial progress towards correct solution.

Year 12 Mathematics Extension 1 Task 4 Examination 2019

Question No.12 Solutions and Marking Guidelines

Outcomes Addressed in this Questions

H2 - constructs arguments to prove and justify results

PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

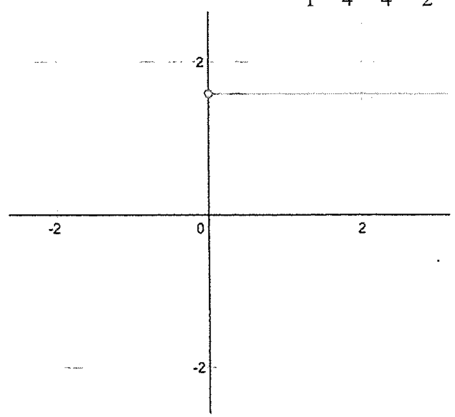
HE3 - uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion or exponential growth and decay.

PE5 - determines derivatives which require the application of more than one rule of differentiation

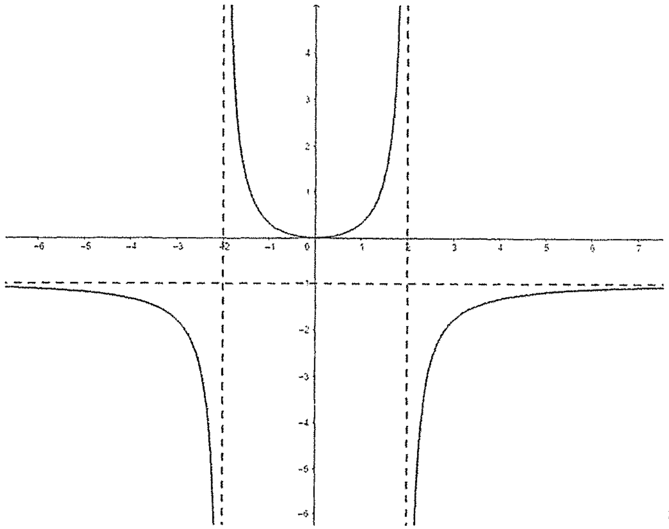
HE6 - determines integrals by reduction to a standard form through a given substitution.

H6 - uses the derivative to determine the features of the graph of a function

HE 7 - Evaluates mathematical solutions to problems and communicates them in an appropriate form.

Outcomes	**Solutions**	**Marking Guidelines**
PE5	a i) $y = \tan^{-1}u$; $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$, $u = \frac{1}{x}$, $\frac{du}{dx} = -\frac{1}{x^2}$; $= \frac{1}{1+u^2}\times\left(-\frac{1}{x^2}\right)$; $= -\frac{1}{1+x^2}$	**2 marks** Correct solution with appropriate working **1 mark** Error made *note: if answer was inferred from part ii then 1 mark was awarded in part ii and zero for part i
H2	ii) $\frac{dy}{dx}(\tan^{-1}x + \tan^{-1}\frac{1}{x}) = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0$	**1 mark** Correct solution
H6	iii) $y = \tan^{-1}x + \tan^{-1}\frac{1}{x}$ is a constant for $x > 0$ since $\frac{dy}{dx} = 0$ When $x = 1$ then $\tan^{-1}1 + \tan^{-1}\frac{1}{1} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ 	**1 mark** Correct solution

HE6	b) $x = u^2$ $\dfrac{dx}{du} = 2u \qquad x = 1 \to u = 1$ $\qquad\qquad x = 9 \to u = 3$ $dx = 2u\,du$ $\displaystyle\int_1^3 \frac{2u\,du}{u^2+u} = \int_1^3 \frac{2\,du}{u+1}$ $= \left[2\ln(1+u)\right]_1^3$ $= 2(\ln 4 - \ln 2)$ $= 2\ln 2$ $= \ln 4$	**3 marks** Correct solution **2 marks** Error made **1 mark** Correctly changed integral limits
PE3	c) $P(1) = 2 - 5 - 9 + 18 = 6$ $P(2) = 16 - 20 - 18 + 18 = -4$ A root lies between $1 < x < 2$ since $P(x)$ is continuous and $P(1)$ and $P(2)$ have opposite signs.	**2 marks** Correct solution with appropriate reasoning **1 mark** Error made or reasoning incomplete
HE3 H2	d i) $N = 125 + Ae^{kt}$ $\dfrac{dN}{dt} = Ake^{kt}$ $= k(125 + Ae^{kt} - 125)$ $= k(N - 125)$	**2 marks** Correct solution with appropriate reasoning **1 mark** Error made or reasoning incomplete / missing steps
HE3	ii) $25650 = 125 + Ae^0$ $A = 25525$ so $N = 125 + 25525e^{kt}$ $31100 = 125 + 25525e^{5k}$ $\dfrac{1239}{1021} = e^{5k}$ $\ln\dfrac{1239}{1021} = 5k\ln e$ $k = \dfrac{\ln\frac{1239}{1021}}{5} = 0.039$	**2 marks** Correct solution **1 mark** Error made

HE7	e) 	**2 marks** Correct solution, including smooth graph and correct behaviour of curves at asymptotes **1 mark** Error made

Year 12 2019	Mathematics Extension 1	HSC Task 4-Trial Examination
Question 13	**Solutions and Marking Guidelines**	
	Outcomes Addressed in this Question	

HE2 - uses inductive reasoning in the construction of proofs
H2 - constructs arguments to prove and justify results
HE3 - uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion or exponential growth and decay.
HE 7 - Evaluates mathematical solutions to problems and communicates them in an appropriate form.

Outcome	Solutions	Marking Guidelines
HE2	a) Step 1: Test the statement for $n=1$ $9^{n+2}-4^n=729-4=725$ this is divisible by 5. Step 2: Assume the statement is divisible by 5 for $n=k$ Thus $9^{n+2}-4^n=5M$ where M is a positive integer. Step 3: Prove the statement is divisible by 5 for $n=k+1$. $9^{k+3}-4^{k+1}=9(9^{k+2})-9(4^k)+9(4^k)-4(4^k)$ $=9(9^{k+2}-4^k)+4^k(9-4)$ $=9(5M)+5(4^k)$......................from step 1 $=5(9M+4^k)$ $=5N$ where N is a positive integer. Hence if the statement is divisible by 5 for $n=k$, it is also divisible by 5 for $n=k+1$. It is divisible by 5 for $n=1$, so it is divisible by 5 for $n=2$. If it is true for $n=2$ it is also true for $n=3$ and so on. It is then true for all positive integers, n.	3 marks: complete correct proof 2 marks: substantial work that could lead to a correct proof 1 mark: correct working for step 1
HE7, H2	b) (i) $(a+b)^n-(a-b)^n=\left[a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\binom{n}{3}a^{n-3}b^3+...+b^n\right]$ $-\left[a^n-\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2-\binom{n}{3}a^{n-3}b^3+...+(-1)^n b^n\right]$ $=2\left[\binom{n}{1}a^{n-1}b+\binom{n}{3}a^{n-3}b^3+\binom{n}{5}a^{n-5}b^5+...\right]$ $\therefore (a+b)^n-(a-b)^n=2\binom{n}{1}a^{n-1}b+2\binom{n}{3}a^{n-3}b^3+2\binom{n}{5}a^{n-5}b^5+...$	2 marks: complete correct show 1 mark: for substantial work that could lead to a correct show
HE7, H2	(ii) If n is even, $(-1)^n b^n$ is positive, $\therefore$ last term is $2\binom{n}{n-1}ab^{n-1}=2nab^{n-1}$.	1 mark: correct last term if n is even
	c)	
HE3	i) $\left(\frac{5}{6}+\frac{1}{6}\right)^n$ is the binomial probability, since probability of a six $=\frac{1}{6}$. $\therefore$ Probability of exactly r sixes, $P(r)=\binom{n}{r}\left(\frac{5}{6}\right)^{n-r}\left(\frac{1}{6}\right)^r$	2 marks: complete correct solution 1 mark: substantial work that could lead to a correct solution
HE3	ii) Probability of an odd number of sixes $=P(1)+P(3)+P(5)+...$ $=\binom{n}{1}\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)+\binom{n}{3}\left(\frac{5}{6}\right)^{n-3}\left(\frac{1}{6}\right)^3+...$ $=\frac{1}{2}\left[\left(\frac{5}{6}+\frac{1}{6}\right)^n-\left(\frac{5}{6}-\frac{1}{6}\right)^n\right]$, from b) $=\frac{1}{2}\left[1-\left(\frac{2}{3}\right)^n\right]$	2 marks: complete correct solution 1 mark: for substantial work that could lead to a correct solution
HE7	d) $A(-3,7)$ $B(1,6)$ $-1:2$ P has coordinates $\left(\frac{(2)(-3)+(-1)(1)}{2-1},\frac{(2)(7)+(-1)(6)}{2-1}\right)$ $=(-7,8)$	1 mark: for x coordinate 1 mark: for y coordinate
HE7	e) We are given $\frac{dr}{dt}=-8$ and we wish to find $\frac{dV}{dt}$ $V=\frac{4}{3}\pi r^3$ $\frac{dV}{dr}=4\pi r^2$ $\therefore \frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ $=4\pi r^2(-8)$ $=-32\pi r^2$ and when $r=100$ $\frac{dV}{dt}=-32\pi(100)^2$ $=-320\,000\pi\ \text{mm}^3/\text{s}$	3 marks: complete correct solution 2 marks: for finding $\therefore \frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ $=4\pi r^2(-8)$ 1 mark: for finding $\frac{dV}{dr}=4\pi r^2$

Year 12 Mathematics Extension 1 Trial 2019
Question No. 14 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE2 uses multi-step deductive reasoning in a variety of contexts
PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Outcome	Solutions	Marking Guidelines
PE3	**(a)** (i) $4C2\times 3C2=18$	**Award 1** mark for the correct solution.
	(ii) $3C1\times 2C1=6$	**Award 1** mark for the correct solution.
PE2	**(b)** **(i)** **Given** $BC=CD$ ΔBCD **is Isosceles triangle (two equal sides in a triangle)** $\angle DBC=\angle BDC=\alpha$ **(angles opposite equal sides in an Isosceles triangle are equal)** $\angle DCR=2\alpha$ **(exterior angle equals to the sum of interior opposite angles)**	**Award 1** mark for the correct solution.
PE2	**(ii)** $\angle BAD \therefore = \angle DCR$ **(exterior angle equals to the opposite interior angle in cyclic quadrilateral** $BADC$**)** $\therefore \angle BAD=2\alpha$ $\because AO$ **bisects** $\angle BAD$ $\angle OAD=\alpha$	**Award 1** mark for the correct solution.
PE2	**(iii)** $\angle TAO=90^0$ **(angle between the tangent and radius at the point of contact)** $\angle TAD=90^0-\alpha$ **(adjacent anlge sum)** $\angle ABD=90^0-\alpha$ **(Alternate Segment Theorem)** $\angle ABC=\angle ABD+\angle DBC=90^0-\alpha+\alpha=90^0$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.
PE3	**(c)** $\alpha+\beta+\gamma=-\frac{8}{3}$ $\alpha\beta+\beta\gamma+\alpha\gamma=\frac{0}{3}=0$ $\alpha\beta\gamma=-\left(\frac{-1}{3}\right)=\frac{1}{3}$ $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\gamma}\right)\left(\gamma+\frac{1}{\alpha}\right)$ $=\left(\alpha\beta+\frac{\alpha}{\gamma}+1+\frac{1}{\beta\gamma}\right)\left(\gamma+\frac{1}{\alpha}\right)$ $=\alpha\beta\gamma+\beta+\alpha+\frac{1}{\gamma}+\gamma+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\alpha\beta\gamma}$ $=\alpha\beta\gamma+\frac{1}{\alpha\beta\gamma}+\alpha+\beta+\gamma+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ $=\alpha\beta\gamma+\frac{1}{\alpha\beta\gamma}+\alpha+\beta+\gamma+\frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha\beta\gamma}$ $=\left(\frac{1}{3}\right)+\frac{1}{\left(\frac{1}{3}\right)}+\left(-\frac{8}{3}\right)+\frac{0}{\left(\frac{1}{3}\right)}=\frac{1}{3}+3-\frac{8}{3}=\frac{2}{3}$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.
PE3	**(d)** **(i)** The equation of the normal to the parabola at P and Q is given by the equations $x+py=ap^3+2ap$ and $x+qy=aq^3+2aq$ respectively. Solving simultaneously $x+py=ap^3+2ap\to A$ $x+qy=aq^3+2aq\to B$ $A-B$ $(p-q)y=a(p^3-q^3)+2a(p-q)$ $(p-q)y=a(p-q)(p^2+pq+q^2)+2a(p-q)$ $y=a(p^2+pq+q^2)+2a$ $\therefore y=a(p^2+pq+q^2+2)$ **Sub** $y=a(p^2+pq+q^2+2)$ **in** A $x+pa(p^2+pq+q^2+2)=ap^3+2ap$ $x=ap^3+2ap-pa(p^2+pq+q^2+2)$ $x=ap^3+2ap-ap^3-ap^2q-apq^2-2ap$ $\therefore x=-apq(p+q)$ $\therefore R\left(-apq(p+q),a(p^2+pq+q^2+2)\right)$	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the correct solution.
PE3	**(ii)** $x=-apq(p+q)$, **given** $pq=-2$ $x=2a(p+q)$ $(p+q)=\frac{x}{2a}$ $y=a(p^2+pq+q^2+2)$ $y=a(p^2-2+q^2+2)$ $y=a(p^2+q^2)$ $y=a\left((p+q)^2-2pq\right)$ $y=a\left(\left(\frac{x}{2a}\right)^2-2(-2)\right)$ $y=\frac{x^2}{4a}+4a$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.

Q1 B

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ (rearranging $cos2x = 1 - 2sin^2x$)

$\int 2\sin^2 x \ dx$

$= \int 2(\frac{1}{2}(1 - \cos(2x))$

$= \int (1 - \cos(2x))\, dx$

$= \int 1 \ dx - \int \cos(2x)\, dx$

$= x - \frac{1}{2}sin2x + c$

Q2. D

$cos\theta = -\frac{1}{2}$

$= \cdots, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \ldots$

or $\theta = 2\pi n \pm \cos^{-1}\left(-\frac{1}{2}\right) = 2\pi n \pm \left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right)$

$= 2\pi n \pm \left(\pi - \frac{\pi}{3}\right)$

$\therefore 2n\pi \pm \frac{2\pi}{3}, where\ n\ is\ an\ integer$

Q3. B

Q4 D

$y = x^3 + 2$

swap the varibles $(x$ *and* $y)$

$x = y^3 + 2$

Solve the equation for y

$y^3 = x - 2$

$y = \sqrt[3]{x - 2}$

Q5. A

Binomial expansion of $(a + y)^4$

$= a^4 + 4a^3y + 6a^2y^2 + 4ay^3 + y^3$

When $a = 2x, y = 3$

$= 16x^4 - 96x^3 + 216x^2 - 216x + 81$

$\therefore -96$

Q6 C

P(2F)=5C2 $(0.48)^2(0.52)^3$

$= combinate\ 2F3M \times 2\ Female \times 3\ males$

Q7 B

Sub in $x = 1$ to binomial expansion given

$\therefore \sum_{r=0}^{n} nCr = 2^n$

Q8. A

$P(-2) = -8 + 4a + 7 = 11$

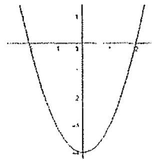
$4a - 1 = 11$

$a = 3$

Q9. A

$\frac{x^2-4}{2x} < 0$

$(x + 2)(x - 2) < 0$



$0 < x < 2$

Q10. B

$5! \times 4! = 2\ 880$